***The Ideal Non-euclidean Gyroscope*: an unresolved problem**
How best to transform coordinate axes in Non-euclidean Geometries?

James D. Barnes (Jim Barnes); Dallas, Texas, USA 13th July 2021, last revised 18th July 2021

In Euclidean geometry we have standard procedures for transforming coordinate axes from any given "point of origin" to any other given position within the space. The mutually perpendicular axes can slide around ("be translated") in a strictly parallel manner, always returning to their original position keeping exactly their same alignment – much like a spinning gyroscope preserves its axial alignment. Further procedures may rotate the axial coordinates. The beautiful feature of all these operations is that the axes always return to their starting position without any unexpected change. We don't get lost or disoriented.

Non-euclidean spaces don't work like this – the "parallel postulate" of euclidean geometry is nullified, and with parallel motion being impossible, the gyroscopic axis system tends to return to its starting position with an altered orientation. A standard test might be to move the perpendicular coordinate axes through a circuit of three arbitrarily selected sites, returning to its beginning without loss of direction. How can we preserve the orientation of the axes through these multiple transformations in Non-eculidean Geometries?

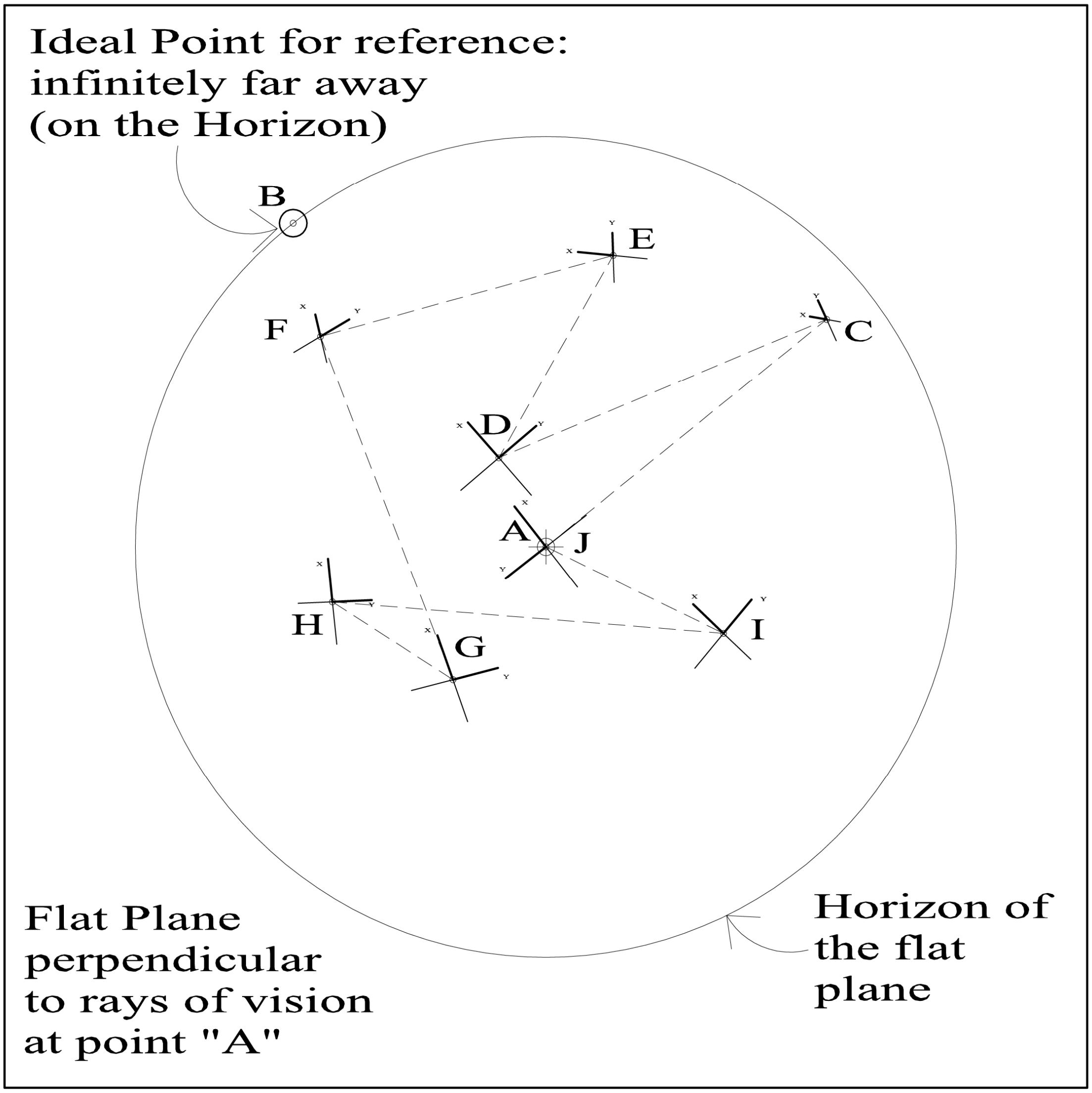
I like the physical imagery of the gyroscope but I am also aware that in a physical non-euclidean space (such as one predicted by the Theory of General Relativity), a real gyroscope (typically) does not return to its original position with its exact same orientation (as the measurable precession of the orbit of spinning planet Mercury physically demonstrates). I am not seeking this physical gyroscope, but an Ideal Gyroscope. My use of the word "gyroscope" is purely metaphorical. The physical gyroscope is a different problem.

It turns out that I can see several different possible solutions. Which method might be best? Perhaps several different methods might be useful to have. I have studied this problem, and I am confident that a better solution (or solutions) can be formulated.

This is basic building operation in any geometry – basic navigation. Having procedures standardized, proven, and available might help future exploration of Non-euclidean spaces.

One set of possible axis transformation methods goes like this:
In Elliptic (Riemannian) a co-ordinate axes may be slid around a flat plane by arbitrarily assigning a single given point as its "pole star" to which one axis will always point. On a flat plane in Hyperbolic (Lobachevskian) Geometry, we may similarly arbitrarily select any "Real point", "Ideal point", or "Ultra-Ideal point" . To expand these methods into three-dimensional spaces, a single flat plane may arbitrarily assigned be as a reference orientation.

With several different methods being possible, the search for the "best" method might include computational considerations, including the ease of spotting computation error.



Hyperbolic $k= 40$ $OA= 80$

Bibliography:
Slawjanowski, Jan J. "Deformable Gyroscope in a Non-Euclidean Space -- Classical, Non-Relativistic Theory", Reports on Mathematical Physics, Volume 10 (1976), Number. 2, pages. 219-243.

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