The Non-euclidean Optics of Euclid

by Jim Barnes; Oklahoma City *3rd January 2014, revised thru 9th February 2014*

In "**Comments on *The Optics of Euclid*"** [1] I wrote:

> "In his ***Optics*** Euclid does not express angles in terms of magnitudes, but rather in terms of 'greater than', 'less than', or 'equal to'. The exact structural interrelationship of the angles is rather loose. It is so loose, in fact, that I can transpose from euclidean to non-euclidean geometry and start reading the text as '***The Non-euclidean Optics of Euclid***'. Such an interpretation requires redefining such words as 'ratio' between similar triangles, 'square', and, of course 'parallel'. I will not try to pretend that the author intended such a skewed reading, but I do think that perhaps Euclid was trying to fashion the text so that future theorists could account either for what we pronounce optical illusions or what we might term "non-symmetries and irregularity of the retinal plane." [2]

Now I feel prepared to be bolder and hypothesize (*again* [3]) that Euclid was consciously exploring non-euclidean geometries in his ***Optics***, generalizing beyond the restrictions of the 4th and 5th Postulates (of his ***Elements***).

About Euclid's life I would like to add [4] two remarks. First, the best near-contemporary "fact" coming down to us is that Euclid had "pupils" [5]. And secondly, I find it striking that, while his name supposedly means "*good reputation*", Euclid's mathematical accomplishments generated no personal reputation. Like many other, I see Euclid as a collector-editor -- his ***Element*s** merely another in a long history of such anthologies, and his ***Optics*** a similar scrapbook collection. While it is possible that Euclid's writings were composed in a single short period, I more readily imagine them as works refined through many years of changes. Possibly they were never "completed". It is likely that the "standard versions" we have today were further edited and extended.

The history of Non-euclidean Geometry is an interesting saga. In the opening of several accounts I've seen the quip: *Who was first to discover Non-euclidean Geometry? Euclid -- because he was the first to appreciate the difference*. Why should we believe that Euclid was incapable of grasping a larger field of non-euclidean (pre-euclidean) logic?

This new hypothesis, that ***The Optics*** is about Non-euclidean Geometry, requires that we assume that Definitions and Postulates of ***The Elements*** do not necessarily apply in ***The Optics***. This new hypothesis might help answer the questions: What is ***The Optics*** about, and why is its logic so "loose"? Euclid could have alternatively refined the art of Linear Perspective, expanding on the now-lost Greek treatises cited by Vitruvius.[6] Or Euclid could have built upon optical observations of Aristotle[7] and modeled the *camera obscura*.

In exploring such a hypothesis one should not lose sight of a belief that the thrust of Euclid's work is to distill euclidean principles. Just as ***The Elements*** starts its theorems without using the 5th Postulate and only slowly added that refinement toward a euclidean goal, so too might read ***The Optics*** as starting in a non-euclidean mode (a pan-geometry) and focusing into a euclidean vision. Euclid's readers were all, in some sense or another, non-euclidean geometers and he is gently showing them that euclidean principles are a common ground upon which all may stand. In modern times, it is hard for those of us raised on the strict rules of euclidean geometry now to see the pre-euclidean possibilities through which Euclid would, from necessity, have freely maneuvered.

Given the uncertainties of books of such antiquity, I foresee great difficulty in *proving* (beyond reasonable doubt) this new hypothesis about ***The Optics***. What may be the hypothesis's greater value is not so much in proving that Euclid held a mastery of Projective Geometry that permitted him to see beyond his 4th and 5th Postulates, but in rediscovering that we ourselves might use Perspective as such a tool.

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[1] Referring to: Euclid, **Optics** (c. 300 B.C.); edited into Latin and Greek by philologist J.L.Heiberg (1895); translated into English by Harry Edwin Burton for publication in the **Journal of the Optic Society of America,** *Vol.* ***35, No.5*** (May, 1945); pp. 357-372.
A complete reprint of this English translation is included with my 1990 "**Comments**" (cited in footnote 2).
There is now also an English language translation available of the Arabic manuscripts:
Elaheh Kheirandish, **"The Arabic Version of Euclid's *Optics*"**; Volumes I and II; Springer; 1999.

[2] James D. Barnes, "**Comments on the '*Optics of Euclid*'**"; [private / Getty library] ;(1990), p. 13.

[3] James D. Barnes, "**Perspective: A Personal Archive: 1991**", [private only] p. 101.

[4] "**Comments**" (op. cit.), pp. 3-5; and various private fantasies in "**Archive: 1991**" (op cit.), pp. 111-113.

[5] Heath, T.; "**A History of Greek Mathematics, Vol. I**", (Dover, p. 356) citing: **Pappus**, vii, p.678, 10-12.

[6] Vitruvius, "**The Ten Books on Architecture**"; VII, 11; (Dover Publications, 1960 etc., p. 198).

[7] Aristotle, "**Problems**", XV, 6 and 11.